

Original article

The Impact of Parental Mathematical Expectations on Young Children's Number Sense: The Serial Mediating Role of Parental Mathematical Involvement and Children's Self-Regulation

Yiwei Zheng, Yi Lv, Jiejiao Li, Jing Zhou*

College of Teacher Education, Ningbo University, Ningbo 315211, P. R. China

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Abstract: Number sense is regarded as the fundamental basis for young children's formal mathematical learning. Grounded in the Family Cultural Capital Theory, this study investigated the effects of parental expectations, parental involvements and self-regulation on the number sense of 847 young children in a city of Eastern China. The results indicated that: (1) parental mathematical expectations, mathematical involvement, and children's self-regulation directly influenced young children's number sense; (2) parental mathematical involvement mediated the relationship between parental expectations and number sense; and (3) parental mathematical involvement and children's self-regulation sequentially mediated the relationship between parental mathematical expectations and children's number sense. The findings suggested that parents should maintain positive mathematical expectations while simultaneously fostering the development of children's self-regulation and number sense by increasing high-quality parental mathematical involvement, particularly through informal mathematical games.

Keywords: parental mathematical expectations; parental mathematical involvement; self-regulation; number sense; young children

1. Introduction

Number sense refers to young children's fluidity and flexibility with numbers, their perception of number meanings, as well as their ability to perform mental calculations, observe the world, and make comparisons (Gersten & Chard, 1999). It primarily consists of five components: counting, number knowledge, number estimation, number transformation, and number patterns (Berch, 2005). As a form of informal mathematical ability (Ginsburg et al., 2008), number sense is the prerequisite for formal mathematical capability and serves as a significant predictor of future academic achievement (Nguyen et al., 2016). Number sense improves gradually with age and demonstrates a certain cumulative effect (Liang, 2020).

Ecological Systems Theory posits that individuals live within a complex network of ecological systems, and their behavioral and psychological development is influenced by the interplay of internal and external factors (Bronfenbrenner, 1986). As a key internal factor, self-regulation helps young children to manage their focus and behavior during math tasks. Children with stronger self-regulation performed better in processing complex math information (Blair et al., 2007). On the other hand, parental mathematical expectations (PME) and parental mathematical involvement (PMI) are vital external factors. PME reflects the goals parents set for their children, while PMI refers to the actual math activities parents do at home. High PME sets a positive tone for learning, and frequent PMI provides children with direct practice (DeFlorio, 2011; Zhang et al., 2019). Together, these internal and external factors create the foundation for number sense development. Most studies examined these factors separately. There is little research on how these internal and external factors work together and their effects on young children's number sense. The present study aimed to examine a serial mediation model which explored how PME related to young children's number sense and tested the mediating roles of PMI and self-regulation.

1.1 The Impact of Parental Mathematical Expectations (PME) on Children's Number Sense

PME is a core part of the home mathematics environment. It acts as a developmental benchmark for children's math growth (Skwarchuk et al., 2014). These expectations consist of two main parts. The first part involves normal expectations. These goals align with typical math skills for a child on certain age. The second part involves high expectations. These goals go beyond standard developmental milestones (Cheng et al., 2023). According to Family Cultural Capital Theory, these expectations serve as a form of "cultural capital" (Richardson, 1986). They significantly shape a child's academic performance and act as strong predictors of early math ability (Kleemans, 2012; Pan et al., 2019).

PME influences number sense through various pathways, but research results are mixed. Some studies show that children from high expectation families have stronger mathematical ability and better learning engagement (Musun - Miller & Blevins - Knabe, 1998). They also perform better than their low expectation peers in number sense tasks (Huang & Cheng, 2011; DeFlorio & Beliakoff, 2015). Align with previous research, DeFlorio (2011) suggests that parents should hold high PME and accurately understand the process of mathematical learning, which are crucial for the successful learning results for young children. However, other research reports no direct correlation (Missall et al., 2015). This suggests that PME may affect number sense through other variables. For instance, expectations for complex number sense skills often require home math activities to improve learning outcomes (Yang et al., 2019). Huang et al. (2017) also found that mothers and fathers play different roles during mathematical learning. Their PME leads to different types of rearing behavior, which then affect young children's mathematical learning.

Most studies look at PME and number sense as a simple, direct relationship. Very few researchers explore how PME works in a chain with other factors. There is a need to investigate how these beliefs turn into specific actions and internal child skills to support math development.

1.2 The Mediating Role of PMI and Children's Self-Regulation between PME and Children's Number Sense

PMI refers to how parents participate in their children's math learning at home (Epstein, 2013). It includes formal and informal interactions between parents and young children, which affect young children's mathematical learning (Huang et al., 2017). The frequency of these activities are key indicators of the home math environment (Cheng et al., 2023). As a form of "objectified cultural capital," these activities are vital for the development of early number sense (Richardson, 1986). Since children develop basic math sensitivity before starting school, these home practices are very important for early learning (Starkey, 2000).

PMI is a key reason why children develop strong number sense. Research shows that Chinese children perform better in math than Western children because Chinese parents engage in more home learning activities (Cheung & Pomerantz, 2011; Huntsinger et al., 2000). This link between PMI and math skills remains strong even after controlling children's gender, language, and intelligence (Niklas et al., 2014). Longitudinal studies confirm that PMI helps children grow their math abilities over time (Wei et al., 2020). Interestingly, young children benefit more from informal activities like games than formal teaching (Dunst et al., 2017). These daily interactions set up a solid foundation for young children to better understand numbers.

PME function as a primary antecedent to PMI, modulating both the frequency and the quality of parental engagement. Parents with high expectations organize math activities more frequently (Susperreguy et al., 2020). They influence young children with home-based mathematical activities such as formal arithmetic practice and math-related reading (Chen, 2018; Skwarchuk et al., 2014). PME also affect the way mothers and fathers interact with their children. Mothers often focus on basic skills like counting, while fathers engage in more games and daily life activities, such as discussing prices while shopping (Huang et al., 2017). These different behaviors help children learn math in various ways. Previous studies have already confirmed a mediation path: parental mathematical expectations affect parental mathematical involvement in a family, which further affect young children's number sense. This confirms that parental beliefs shape the home environment, which further affect young children's academic performance (LeFevre et al., 2010).

Self-regulation is a key factor of children's number sense development (Xia & Cao, 2017). It is the process children use to manage their emotions, behaviors, and thoughts to adapt to different situations (Blair & Diamond, 2008). Following the method outlined by Wang (2024), self-regulation was measured using two different approaches: "cool" and "hot" regulation. "Cool" self-regulation refers to children's performance on cognitive tasks in neutral or low-emotion situations. In contrast, "Hot" self-regulation refers to the performance in tasks that require emotional or motivational control. Both hot and cool self-regulation are needed during young children's learning because they help young children to better control their attention, thoughts, and behaviors (Ívrendi et al., 2016).

The home environment plays a large role in shaping a child's self-regulation. During early childhood, parents often act as "external regulators" that help young children to better manage their behavior (Schunk & Zimmerman, 1997). Through consistent parental involvement, children gradually learn how to regulate their own thoughts and actions. Positive involvement from parents predicts better self-regulation in children (Polat & Bayındır, 2020). Parental beliefs also matter. As for early mathematical learning, researches have found that excessively high PME can actually impedes the development of a child's self-regulation (Bechtel-Kuehne et al., 2016; Zhu, 2024). This suggests that PME and PMI might influence number sense by first affecting how well a child can regulate themselves.

Most previous studies investigate the internal and external factor that affect young children's number sense separately. There is no research that combines these two kinds of factors into one model. This study try to investigate the relationships between these two kinds of factors and young children's number sense. To be specific, this study aims to reveal whether PME would affect PMI, which would further affect young children's self-regulation, and whether this relationship will affect young children's number sense finally.

1.3 The Current Study

The purpose of this study was to explore the relationships among parental mathematical expectations, parental mathematical involvement, child self-regulation, and child number sense. Based on the literature review, a hypothesized model was constructed (see Figure 1). In this model, parental mathematical expectations (PME) serves as the independent variable, child number sense as the dependent variable, and parental mathematical involvement (PMI) and child self-regulation as mediating variables. The specific hypotheses proposed in this study are as follows: (1) PME, PMI, and child self-regulation had direct effects on young children's number sense; (2) PMI mediated the relationship between PME and young children's number sense; (3) PMI and child self-regulation played a serial mediating role in the relationship between PME and young children's number sense.

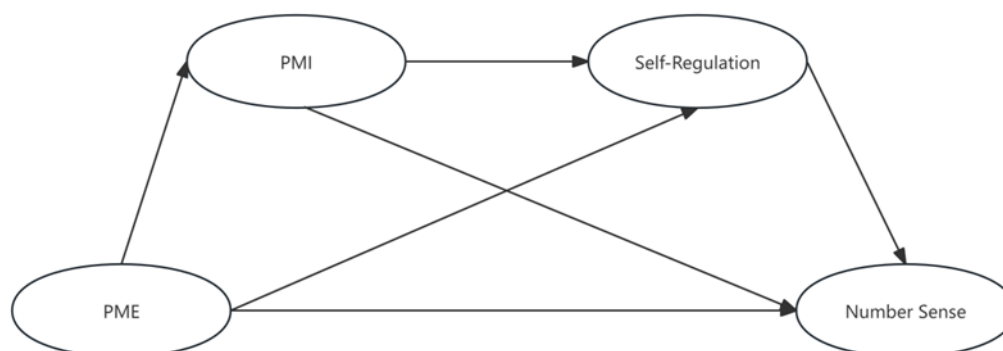


Figure 1. Hypothesized Serial Mediating model

2. Method

2.1 Participants

Participants were recruited from nine kindergartens in an Eastern city of China. A total of 925 questionnaires were distributed to parents. 78 cases were excluded because of incomplete parental responses or invalid child assessment data. This process resulted in a final sample of 847 parent-child dyads. The effective response rate for the study was 91.57%. The final sample comprised 425 boys ($M = 52.68$ months, $SD = 6.27$) and 422 girls ($M = 52.57$ months, $SD = 5.86$). Additionally, 56% of the participating families were from rural area.

2.2 Measures

2.2.1 Parental Mathematical Expectations (PME)

This study employed the Chinese version of Parental Mathematical Expectations Questionnaire (Cheng et al., 2023) to assess PME. The 9-item questionnaire was developed to measure parental view of their young children's development benchmarks before entering primary school, which consisted two sections: (a) reasonable PME, which assess developmentally appropriate milestones (e.g., "Recognizing numbers within 100"), and (b) over-high PME, which encompass skills that typically exceed preschool capabilities (e.g., "Solving addition and subtraction problems within 100"). Each item was rated on a 5-point Likert-type scale ranging from 1=*not important at all* to 5=*extremely important*. The average scores of the nine items were calculated with higher scores indicating higher PME. The scale demonstrated robust internal consistency for the total measure (Cronbach's $\alpha = 0.88$) as well as for the reasonable ($\alpha = 0.88$) and overly high ($\alpha = 0.87$) sections in this study.

Parental Mathematical Involvement (PMI)

The study employed the Chinese version of Parental Mathematical Involvement Questionnaire which was developed by Huang et al.(2017) to capture the frequency of home-based numeracy practices. This 21-item questionnaire assesses parent-child mathematical interactions over the preceding month across four distinct dimensions: (a) number skills (e.g., "Identifying names of numbers"); (b) number books (e.g., "Using number activity books"); (c) games (e.g., "Playing card games"); and (d) applications (e.g., "Playing with calculators"). Parents rated the frequency of each activity on a 5-point Likert scale ranging from 1 =*Never* to 5 =*Almost every day*. All ratings were averaged to create a final score. The higher the score, the more involved the parent is in their child's math learning. The Cronbach's α were 0.93 for mathematical skills, 0.92 for number books, 0.89 for games, and 0.89 for applications. In this study, the questionnaire proved to be highly reliable, with an overall Cronbach's $\alpha = 0.91$.

2.2.2 Self-Regulation

Cool self-regulation. This study evaluated young children's cool self-regulation using the Day-Night task (Gerstadt et al., 1994) and Head-Toes task (Ponitz et al., 2008). The Day-Night task assesses if a child can follow a "conflicting" rule instead of just saying the first thing that comes to mind. Researchers showed children 16 cards that featured either a sun with a white background or a moon and stars with a black background. Children were instructed to say "night" when they saw the sun card and "day" when they saw the moon card. After four practice with feedback, researchers conducted 16 test trials. Children earned 1 point for every correct answer. So they could get a maximum score of 16. The Head-Toes

task checks if a child can do the exact opposite of a spoken command. For example, if the instructor said "touch your head", the child was supposed to touch their toes instead. A total of 10 random oral commands were delivered without accompanying physical demonstrations. Researchers used a 3-point scale to grade their performance: 2 points for a correct response, 1 point for self-correction (initiating a wrong movement but self-correcting), and 0 points for an incorrect response, the highest possible score for this task was 20. To get a final "Cool Self-Regulation" score, the study combined the results from both tasks using standardized (z) scores. A higher final score means the child has stronger cognitive and regulatory abilities.

Hot self-regulation. This study measured hot self-regulation using the Gift Delay task (Kochanska et al., 1996). This task tests a child's ability to control their impulses and wait for a reward, even when they are feeling excited. First, researchers promised each child a gift for doing a good job. Then we asked them to turn around and wait for 120 seconds while the researcher wrapped the gift behind them. The researcher wrapped the gift noisily to make it more tempting, and researchers told the children not to peek. The performance was graded based on two ways: (1) Peeking behavior (0 = turning around/peeking twice; 1 = peeking once; 2 = no peeking), and (2) Waiting time (the exact number of seconds elapsed before the first instance of peeking was recorded). Finally, these two metrics were standardized and aggregated into a single "Hot Self-Regulation" score with higher score indicating better ability of hot self-regulation.

2.2.3 Number Sense

The study utilized the Early Number Sense Test to assess early mathematical skills. This test was based on the framework by Malofeeva (2004) and was later adjusted by Zhao (2008) to better fit Chinese populations. The final version of the test includes 21 different tasks and a total of 163 items. These items are organized into six dimensions: counting, number identification, number-object correspondence, ordinality, comparison, and addition-subtraction. The overall reliability score (Cronbach's α) was 0.94. For the six individual math areas, the scores ranged from 0.60 to 0.94, which showed that the test is a very consistent way to measure early number sense.

2.3 Procedures

This study met the ethical norms and standards. First, researchers contacted the preschool directors and obtained their permission for the data collection. Second, researchers communicated with the supervising teacher of each class. With the assistance of the teachers, the parents were explained about the study. The questionnaires were distributed to the parents who were interested to participate in this study. Then, the teachers collected the completed questionnaires from the parents and handed them to the researchers. Children were assessed one by one in a quiet room in their preschool for their self-regulation and number sense data. The average time of the data collection were about 25 minutes for 3-4-year-old children and 35 minutes for children aged 4-5-year-old. Children were allowed to withdraw from the assessment whenever they want.

Prior to the study, six graduate students were trained to ensure the standardization of data collection. For tokens of appreciation, each child was provided with an age-appropriate gift and sent parents a simple assessment report of their child if they want.

2.4 Data Analysis

Missing values were handled using the full information maximum likelihood (FIML) approach implemented in AMOS 28.0, which takes all available data into account when estimating the model parameters (Schafer & Graham, 2002). Quantitative data analysis proceeded through three sequential phases utilizing SPSS 26.0. First, preliminary analyses included descriptive statistics for all demographic and study variables. Second, structural equation modeling (SEM) with AMOS 28.0 was used to evaluate a series of hypothetical models (see Fig. 1) after determining the best fitting model. Several indices were used to determine the model's goodness of fit: the chi-square (χ^2) value and degrees of freedom (df); the Tucker-Lewis index (TLI); the comparative fit index (CFI); the root mean square error of approximation ($RMSEA$), along with a 90% confidence interval (CI); and the standardized root mean squared residual ($SRMR$). Following the criteria established by Hu and Bentler (1999), TLI and CFI values above .90, and $RMSEA$ and $SRMR$ values below .08, were considered indicative of a reasonable model fit.

3. Results

3.1 Preliminary Analyses

Table 1 shows the descriptive statistics and correlations for all study variables. Generally, children got an average score of 99.20 ($SD = 52.45$) on number sense. Parents got a mean score of 3.79 ($SD = 0.63$) on PME and 2.50 ($SD = 0.69$) on PMI.

PME exhibited significant positive associations with both PMI ($r = 0.22, p < 0.001$) and number sense ($r = 0.14, p < 0.001$). Similarly, PMI was positively correlated with cool self-regulation ($r = 0.09, p = 0.007$) and number sense ($r = 0.27, p < 0.001$). Among the family background variables, urban/rural status showed a significant positive link to number sense ($r = 0.11, p = 0.001$). Although gender was unrelated to number sense ($r = -0.01, p = 0.688$), it demonstrated significant associations with both cool ($r = 0.09, p = 0.014$) and hot self-regulation ($r = 0.13, p < 0.001$). To ensure the accuracy of these relationships, gender and urban/rural status were included as covariates.

Guided by contemporary standards for mediation analysis (Hayes, 2018; Zhao et al., 2010), this study prioritized the significance of specific indirect effects over simple bivariate correlations between variables. Under this framework, a significant correlation between the independent variable and the mediator is not a prerequisite for testing mediation,

provided there is a sound theoretical basis for the indirect pathway. Accordingly, although the direct correlation between PME and cool self-regulation was non-significant, this variable was retained in the final analysis as a valid indirect effect was hypothesized to be transmitted through PMI. In contrast, hot self-regulation was excluded from the structural model because it failed to demonstrate the necessary serial associations within the proposed chain—specifically, it showed no significant relationship with either the antecedent (PME) or the primary mediator (PMI)—thereby lacking the empirical foundation required for a serial mediation sequence. Consequently, the final model incorporated PMI and cool self-regulation as the serial mediators.

Table 1. Descriptive Statistics and Correlation Analysis

Variables	<i>M</i> ± <i>SD</i>	1	2	3	4	5	6	7
1. PME	3.79 ± 0.63	-						
2. PMI	2.50 ± 0.69	0.22***	-					
3. Cool Self-Regulation	0.00 ± 1.62	0.03	0.09**	-				
4. Hot Self-Regulation	0.01 ± 0.97	0.02	-0.00	0.13***	-			
5. Number Sense	99.20 ± 52.45	0.14***	0.27***	0.43***	0.12***	-		
6. Gender	1.50 ± 0.50	0.04	0.02	0.09*	0.13***	-0.01	-	
7. Urban/Rural	1.44 ± 0.50	-0.00	0.05	0.03	-0.11**	0.11**	0.04	-

Note. PME = Parental Mathematical Expectations; PMI = Parental Mathematical Involvement.

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$.

3.2 Mediation Model Test

Confirmatory Factor Analysis (CFA) was conducted to evaluate the measurement models (Figure 2) for the latent constructs of PME, PMI, Self-Regulation, and Number Sense. The results showed an excellent fit for the measurement model ($\chi^2/df = 2.625$, $CFI = 0.981$, $TLI = 0.975$, $RMSEA = 0.044$, and $SRMR = 0.034$). All factor loadings were significant at $p < 0.001$, with the standardised factor loadings ranging from 0.55 to 0.92, with the exception of the Day-Night task in the Cool Self-Regulation construct, which had a loading of 0.43. This implied that the latent variables were represented well by their respective indicators (Hair et al., 2019).

Structural Equation Modeling (SEM) was conducted to examine the direct and indirect effects of PME on young children's number sense. This study first tested the relationships between PME, PMI, and self-regulation with children's number sense (direct-effect model). After controlling for covariates, PME ($\beta = 0.12$, $p = 0.020$), PMI ($\beta = 0.17$, $p = 0.001$), and cool self-regulation ($\beta = 0.57$, $p < 0.001$) directly affected children's number sense. The first hypothesis was proved.

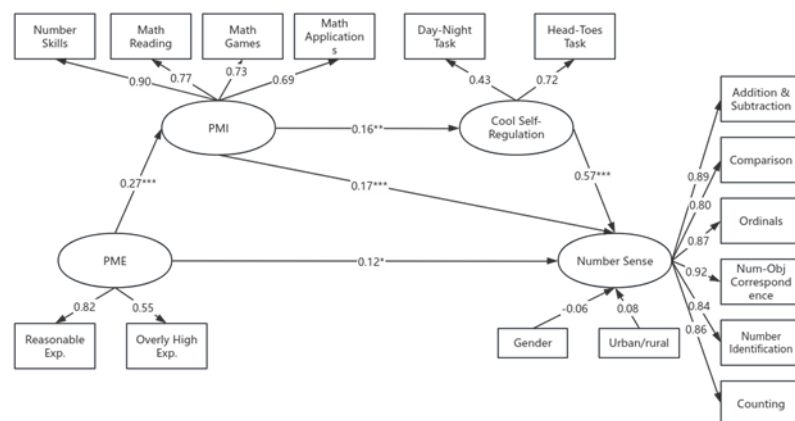


Figure 2. The Serial Mediation Model Linking Parental Mathematical Expectations to Number Sense

In addition to examining the direct effects of PME, this study sought to investigate the mediational pathways by which PME influence children's number sense. PMI and children's cool self-regulation were added to the model to test the mediation effect. Two models were tested with (1) PMI as a single mediator; (2) PMI and cool self-regulation as sequential mediators. The hypothesized serial mediation model demonstrated an excellent fit to the observed data ($\chi^2/df = 2.679$, $CFI = 0.976$, $TLI = 0.970$, $RMSEA = 0.045$, and $SRMR = 0.037$). As shown in Figure 2, the effects of PME on children's number sense were partially mediated by the sequential indirect effects of PMI ($\beta = 0.17$, $p < 0.001$) first, and followed by cool self-regulation ($\beta = 0.16$, $p < 0.001$), which support our hypothesis 2 and 3.

As displayed in Table 2, the analysis confirmed a significant total effect of PME on children's number sense (Estimate = 3.49, 95% CI [1.400, 5.212]). After accounting for the mediators, the direct effect remained significant (Estimate = 2.18, 95% CI [0.192, 3.959]), contributing 62.4% of the total effect. The total indirect effect through PMI and cool self-regulation was 1.31 (95% CI [0.831, 1.917]), which explained 37.6% of the total impact. This total indirect effect consisted

of two significant mediation pathways: The path through PMI was significant (Estimate = 0.84, 95% *CI* [0.451, 1.339]), accounting for 24.0% of the total effect. Furthermore, the serial mediation pathway involving both PMI and cool self-regulation was also significant (Estimate = 0.47, 95% *CI* [0.201, 0.898]), representing 13.6% of the total effect. These results provide strong evidence for the hypothesized serial mediation chain, suggesting that PME influence early numeracy both by increasing PMI and by fostering children's cognitive self-regulation.

Table 2. Decomposition of Direct, Indirect, and Total Effects

Effect	Estimate	SE	95% <i>CI</i>		Proportion of Total Effect
			<i>LL</i>	<i>UL</i>	
Total Effect	3.49	0.96	1.400	5.212	-
Direct Effect	2.18	0.72	0.192	3.959	62.4%
Total Indirect Effect	1.31	0.27	0.831	1.917	37.6%
PME → PMI → Number Sense	0.84	0.22	0.451	1.339	24.0%
PME → PMI → Cool SR → Number Sense	0.47	0.17	0.201	0.898	13.6%

Note. PME = Parental Mathematical Expectations; PMI = Parental Mathematical Involvement; Cool SR = Cool self-regulation. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$.

4. Discussion

The present study investigated the complex mechanisms through which PME influenced early number sense in young children. This research made a unique contribution to the field by testing a serial mediation model that integrated both environmental (PMI) and individual (cool self-regulation) factors. Grounded in the Family Cultural Capital Theory, the results demonstrated that PME predicted children's number sense both directly and indirectly. The findings confirmed that parental beliefs shaped the home mathematics environment and children's cognitive control, which ultimately build the foundation of early mathematical learning.

4.1 The Impact of PME on Young Children's Number Sense

The results revealed connections between PME, PMI, and young children's number sense. First, PME directly influenced children's number sense. It is clear that parental math-related beliefs are strongly associated with children's early cognitive outcomes, a finding that aligns with existing literature emphasizing the importance of parental expectations in early development (DeFlorio, 2011; Kleemans et al., 2012). The direct effect of PME on number sense remained significant even after controlling for the mediating mechanisms of PMI and self-regulation ($\beta = 0.117$, $p = 0.034$).

This direct influence can be explained by the Pygmalion effect (Bronfenbrenner, 1986), which argues the effects of high expectations on children's learning. Parents with high expectations hold more active attitudes towards children's mathematical learning and provide more positive feedback and scaffolding during children's mathematical activities. Children's positive attitudes about math are shaped by their parents high expectations (Cheng et al., 2023), which in turn inspire children to engage more with math tasks. Previous study suggests that parental expectations also change parents' judgement and feedback which further shape children's self-concept of their math abilities and attitudes (Hascoët et al., 2021). The Family Process Model also suggests that PME is a core part of the home math environment (Home Numeracy Environment [HME]; Davis-Kean, 2005), which is positively related to children's mathematical achievement (Cheung et al., 2025).

4.2 The Mediating Role of PMI between PME and Young Children's Number Sense

Mediation analysis revealed the significant mediating role of PMI in the association between PME and number sense, indicating that PME was not only associated with number sense directly, but also indirectly associated with number sense through PMI, which was consistent with literature from Western (Skwarchuk et al., 2014) and Asian nations (Yang et al., 2019). The study confirmed that PMI was a vital link between PME and number sense.

There are several explanations for this mediation role of PMI in the relationship between PME and number sense. On the one hand, within the Chinese culture, children's academic success is a vital family issue. It is common for Chinese parents to hold high expectations for their children's learning, especially mathematical achievement (Luo et al., 2013; Xin & Yu, 2024). It is also prevalent for parents to invest more resources in children's mathematical learning. Previous research found that parents with higher expectations often engage in more home-based mathematical activities (Lefevre et al., 2002; Huang et al., 2022; Cheng et al., 2023).

On the other hand, there are all kinds of home-based mathematical activities provided by high-expectation parents, ranging from activities with didactic ends, such as counting objects, to reading number-related storybooks (Huang et al., 2022; Zhou et al., 2007), to activities that occur in daily life, such as talking about money of number when shopping of playing cards with children (Zhang et al., 2020). According to Ramani and Siegler (2008), children get multi-modal learning experiences from various math-related learning activities. Children develop more advanced mathematical skills, such as problem solving with the help of parents in daily life (Cheung & McBride, 2017). Empirical studies show that parents with high expectations also provide more formal mathematical activities, like arithmetic (Missall et al., 2015), which are very helpful for children's number sense (Del Río et al., 2017).

4.3 The Serial Mediating Role of PMI and Self-Regulation between PME and Children's Number Sense

Sequential mediation analysis revealed the significant mediating role of PMI and cool self-regulation in the association between PME and number sense, indicating a specific step-by-step path. That is, parents with high PME engage more in mathematical activities at home. The high quality parent-child interaction not only affect children's number sense, but also help children build better self-regulation skills. Stronger self-regulation then help children perform better in mathematical learning. This sequential model suggests that children benefit from their parents in two ways. Children develop advanced mathematical skills directly from the home-based mathematical activities. At the same time, they also develop the general cognitive abilities, such as executive function, which significantly promote children's mathematical abilities (Bukva & Memisevic, 2025).

Cheng (2023) noted that expectations are the main reason parents spend time and effort on their children's mathematical learning. This motivation leads to not only more frequent math activities but also a more supportive style of interaction, which is very important for the development of children's self-regulation (Koşkulu-Sancar et al., 2023). Math play, in particular, acts as a natural training ground for executive functions (DePascale & Ramani, 2025). For example, children need the ability of inhibitory control to follow the rules. They need working memory to remember the steps of a game. Through these daily math-related activities, children develop the cognitive tools they need in mathematical learning. Empirical study found that children with stronger self-regulation skills, especially cool executive functions like self-control, performed better in early mathematical learning (Blair et al., 2007; Wang et al., 2017). Evidence from Neuroscience also shows this connection. Dehaene (2004) found solving math problems that are math-related and self-control related activate the same parts of the brain. This suggests that math learning and self-regulation might share the same biological function.

In summary, this study found a complete path from parent beliefs to children's cognitive outcomes. These results showed how parents affect their children through home-based activities and daily parent-child interactions. Parents with high expectations should also engage in children's mathematical learning. Children could benefit from both high expectations and involvement from their parents.

4.4 Limitations and Implications

The present study is one of the first to investigate the underlying mechanism of internal and external factors affecting children's number sense by employing sequential mediation model. The findings confirmed the critical role of PME, PMI, and cool self-regulation in the development of early number sense. It also provided evidence to support parents in promoting children's early mathematical learning in Chinese home numeracy environment.

This study had several limitations. First, we recruited participants from a single city of China. Therefore, the sample was not fully representative of all Chinese families. Second, because of the nature of cross-sectional design, present study couldn't find the causal link between variables. Since math skills and self-regulation change over time, longitudinal design would be better to reveal how children's number sense develop with the change of PME and/or PMI, and the effect of development of children's self-regulation on number sense. Third, this study collected data by questionnaire reported by parents, which might be subjective. Multiple methods, such as observations and child interviews should be used to improve the objectiveness of data.

Nevertheless, the present study still yields several implications, both theoretically and practically. First, current study integrated both internal and external factors into a sequential mediation model and revealed the underlying mechanism, which might expand the Home Numeracy Environment model (Skwarchuk et al., 2014). Based on this model, previous researches focused only on the effect of home mathematical environment, including PME or PMI on children's mathematical learning. The present study found that children's self-regulation also plays a crucial role in the relationships among family factors and children's learning. Moreover, PMI served as a mediation role in the relationship between PME and number sense, which was measured by questionnaire that focused on the frequency of various home-based activities including formal and informal activities. This shed light on the way of parental mathematical involvement in China. Since formal learning activities were often used in traditional Chinese families (Huang et al., 2022; Zhou et al., 2007), this study suggested other kinds of math-related activities, such as math-related games and problem solving in daily life. Compared with formal activities, these informal activities could better inspire children's interest in mathematics, which is crucial for children's mathematical learning.

5. Conclusion

This study investigated the complex mechanisms through which PME influence early number sense in young children. The results demonstrated that PME, PMI, and cool self-regulation each exert significant direct effects on children's number sense. Furthermore, the analysis confirmed that PMI serves as a critical bridge, partially mediating the relationship between parental expectations and early numeracy. Notably, a significant serial mediation pathway was established, revealing that PME influences number sense indirectly by first increasing the frequency of PMI, which subsequently fosters children's cool self-regulation. By integrating both environmental factors and individual cognitive characteristics into a unified framework, this research expands the existing Home Numeracy Environment model and highlights the importance of the interaction between home-based activities and children's cognitive control. In conclusion, these findings suggest that for parental beliefs to effectively promote early numeracy, they must be translated into active, supportive mathematical involvement that simultaneously nurtures children's self-regulatory skills, providing a robust foundation for their

successful transition to formal mathematical learning.

Data Availability Statement

The dataset generated and analyzed during the current study is available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

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